# Graduate Seminar in Representation Theory (S4A2)

**Summer Semester 2024**

**Dr. Mustafa Kalafat**

# Representation Theory with applications to Differential Geometry

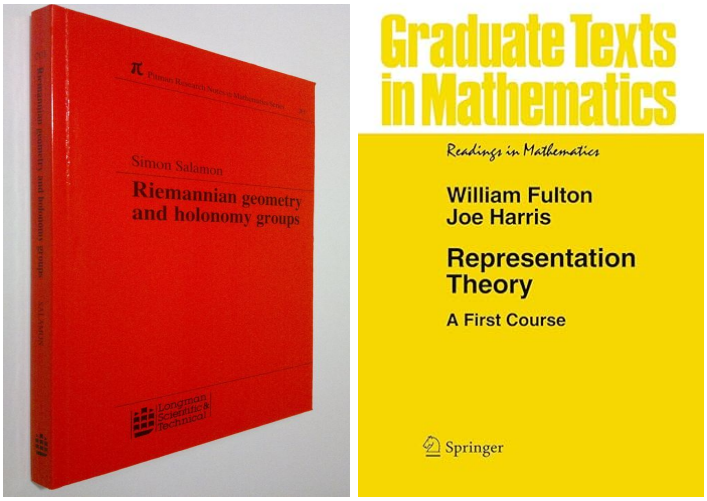


Figure 1: Two foundational texts.

The aim of this seminar is to see the usage of the finite-dimensional representations, mainly over $\mathbb{C}$ of semisimple Lie groups and Lie algebras, with special emphasis on classical groups. We would like to take great care to motivate everything by working through numerous examples and by providing informal proof sketches as well as geometric interpretations and applications wherever possible.

We also study the Lie groups $G_2$ and $Spin(7)$. Understand the representation of their Lie algebras. And see how that effects the geometry of the related manifolds with these structures or holonomy groups.

Students are required to submit a manuscript that displays important aspects of their talks.

If you decide to take this seminar class, send me an e-mail with a preference of at least 3 topics from the syllabus below. Also include your full name, Matricul.no. and Uni-ID in the message.

**Prerequisites:** Advanced geometry 1.
**Organizational Meeting:** February 23 2024, 13:00, in seminar room 0.011.
**Seminar Time and Place:** Wednesdays at 2:15-4 in seminar room 0.011.

**Talks:**

1. Manifolds and structure groups. Parallel transport. Unitary holonomy group. *Ref:* [Sal89] Chapters 1, 2 and 3.

2. Riemannian and Kähler curvature.
   *Ref:* [Sal89] Chapter 4.

3. Lie algebras and symmetric spaces.
   *Ref:* [Sal89] Chapter 5. See also [Kna02, FH91]

4. Representation theory. Weights and Roots. Tensor products.
   *Ref:* [Sal89] Chapter 6. See also [Kna02, FH91]

5. 4 dimensions. Self-dual Einstein manifolds. Twistor spaces.
   *Ref:* [Sal89] Chapter 7.

6. Special Kähler manifolds: Calabi-Yau metrics, Hyperkähler metrics.
   *Ref:* [Sal89] Chapter 8.

7. Quaternionic manifolds and classification theorems.
   *Ref:* [Sal89] Chapters 9 and 10.

8. $G_2$ as a holonomy group.
   *Ref:* [Sal89] Chapter 11.

9. $Spin(7)$ as a holonomy group. Clifford algebras.
   *Ref:* [Sal89] Chapter 12.

10. Representation theory of complex Lie algebras $\mathfrak{sl}(n, \mathbb{C})$.
    *Ref:* [FH91] Chapters 11, 12 and 13.

11. Representation theory of the Lie algebra $\mathfrak{g}_2$.
    *Ref:* [FH91] Chapters 14 and 22.

12. Remarks on $G_2$ structures.
    *Ref:* [Bry06] First 5 sections.

# References


[Bry06] Robert L. Bryant. Some remarks on $G_2$-structures. In *Proceedings of Gökova Geometry-Topology Conference 2005*, pages 75–109. Gökova Geometry/Topology Conference (GGT), Gökova, 2006.

[FH91] William Fulton and Joe Harris. *Representation theory*, volume 129 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 1991. A first course, Readings in Mathematics.

[Kna02] Anthony W. Knapp. *Lie groups beyond an introduction*, volume 140 of *Progress in Mathematics*. Birkhäuser Boston, Inc., Boston, MA, second edition, 2002.

[Sal89] Simon Salamon. *Riemannian geometry and holonomy groups*, volume 201 of *Pitman Research Notes in Mathematics Series*. Longman Scientific & Technical, Harlow; copublished in the United States with John Wiley & Sons, Inc., New York, 1989.

**Assigned Speakers:**

1. Taneski - 2 weeks
2. Taşdemir
3. Cappelli - skip
4. Alejandro
5. Ono
6. Taneski
7. Luca
8. 
9. 
10. Alejandro
11. 
12. Taşdemir - skip